

NAG Toolbox for MATLAB

f07gg

1 Purpose

f07gg estimates the condition number of a real symmetric positive-definite matrix A , where A has been factorized by f07gd, using packed storage.

2 Syntax

```
[rcond, info] = f07gg(uplo, n, ap, anorm)
```

3 Description

f07gg estimates the condition number (in the 1-norm) of a real symmetric positive-definite matrix A :

$$\kappa_1(A) = \|A\|_1 \|A^{-1}\|_1.$$

Since A is symmetric, $\kappa_1(A) = \kappa_\infty(A) = \|A\|_\infty \|A^{-1}\|_\infty$.

Because $\kappa_1(A)$ is infinite if A is singular, the function actually returns an estimate of the **reciprocal** of $\kappa_1(A)$.

4 References

Higham N J 1988 FORTRAN codes for estimating the one-norm of a real or complex matrix, with applications to condition estimation *ACM Trans. Math. Software* **14** 381–396

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo** – **string**

Indicates how A has been factorized.

uplo = 'U'

$A = U^T U$, where U is upper triangular.

uplo = 'L'

$A = LL^T$, where L is lower triangular.

Constraint: **uplo** = 'U' or 'L'.

2: **n** – **int32 scalar**

n , the order of the matrix A .

Constraint: $n \geq 0$.

3: **ap**(*) – **double array**

Note: the dimension of the array **ap** must be at least $\max(1, n \times (n + 1)/2)$.

The Cholesky factor of A stored in packed form, as returned by f07gd.

4: **anorm – double scalar**

The 1-norm of the **original** matrix A , which may be computed by calling Missing 'id'. **anorm** must be computed either **before** calling f07gd or else from a copy of the original matrix A .

Constraint: **anorm** ≥ 0.0 .

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

work, iwork

5.4 Output Parameters1: **rcond – double scalar**

An estimate of the reciprocal of the condition number of A . **rcond** is set to zero if exact singularity is detected or the estimate underflows. If **rcond** is less than *machine precision*, A is singular to working precision.

2: **info – int32 scalar**

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info = $-i$

If **info** = $-i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **uplo**, 2: **n**, 3: **ap**, 4: **anorm**, 5: **rcond**, 6: **work**, 7: **iwork**, 8: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

7 Accuracy

The computed estimate **rcond** is never less than the true value ρ , and in practice is nearly always less than 10ρ , although examples can be constructed where **rcond** is much larger.

8 Further Comments

A call to f07gg involves solving a number of systems of linear equations of the form $Ax = b$; the number is usually 4 or 5 and never more than 11. Each solution involves approximately $2n^2$ floating-point operations but takes considerably longer than a call to f07ge with one right-hand side, because extra care is taken to avoid overflow when A is approximately singular.

The complex analogue of this function is f07gu.

9 Example

```
uplo = 'L';
n = int32(4);
ap = [2.039607805437114;
```

```
-1.529705854077835;  
0.2745625891934577;  
-0.04902903378454601;  
1.640121946685673;  
-0.2499814119483738;  
0.6737303907389101;  
0.7887488055748053;  
0.6616575633742563;  
0.5346894269298685];  
anorm = 10.16;  
[rcond, info] = f07gg(uplo, n, ap, anorm)  
  
rcond =  
    0.0103  
info =  
      0
```
